J. Fluid Mech. (1971), vol. 49, part 1, pp. 206–208
Printed in Great Britain

REVIEWS

Mécanique des Milieux Continus. By J. MANDEL. Gauthier–Villars, 1966 Vol. I, 464 pp., 96 F. Vol. II, 392 pp., 84 F.

Mécanique des Milieux Deformables. By G. GONTIER. Dunod, 1969. 616 pp. 64 F.

The subjects of fluid and solid mechanics have so much in common at the fundamental level that there is much to be said for the approach adopted in both of these textbooks of treating both together under some such general title as 'Mechanics of Deformable Media'. Under this heading, it is possible to treat the Lagrangian and Eulerian descriptions of the motion of a continuum, the equation of mass conservation, deformation and the strain and rate-of-strain tensors, the stress tensor, the Cauchy momentum equation, and the energy equation in a general form; and in this treatment the distinction between solids and fluids need, from an extreme point of view, be no more prominent than the distinction between liquids and gases in low Mach number fluid dynamics. Only at the inevitable point at which the constitutive relation is introduced need the physical distinction between the solid and fluid state be discussed and given analytic expression; from this point on the two main branches of solid and fluid mechanics follow divergent paths, with only occasional opportunity for further natural cross-links and references (e.g. the analogy between slow viscous flow and static deformation in two dimensions, both described by the biharmonic equation). The approach does of course require a certain familiarity on the part of the students with the methods of Cartesian tensor analysis; but when a course on vector and tensor analysis has already been taught at an early stage of an undergraduate's career, as is the practice in many universities, the subject of general continuum mechanics then provides perhaps the best physical context in which the power of tensor methods can be demonstrated.

Mandel's work, in two volumes, is specifically designed to cover the course in the Mechanics of Deformable Media taken by students in their second year at the École Polytechnique in Paris. The first volume is divided into two parts, Part I on generalities, applicable to both solid and fluid mechanics, and Part II on fluid mechanics alone. The second volume contains Part III on the mechanics of solids. The volumes are clearly intended to be taken together, since the index and bibliography to both occurs only at the end of volume II. (The contents also come at the end of the volumes, which takes some getting use to!) Part I starts off with a fluent recapitulation of the theory of second-rank Cartesian tensors, and then goes on to give a thorough treatment of deformation, stress, energy, and thermodynamic relations; there is also an interesting discussion of surfaces of discontinuity in a general continuum. Part II starts with a chapter on hydrostatics, and follows with the theory of ideal fluids, incompressible flow with and without vorticity, sound waves and gravity waves, compression and rarefaction waves, shock waves, and steady compressible flow. Only at p. 292 does the author turn to viscous effects, with a chapter on simple viscous flows, boundary layers and a qualitative discussion of some of the effects of turbulence; and finally there is a chapter on dynamical similarity. Many of the more difficult topics are relegated to appendices; these are evidently not part of the course as taught, but are included for ease of reference. Here, for example, we find such topics as aerofoil theory, free-streamline theory, Stokes flow past a sphere, Hele–Shaw flow, and (what seems rather out of place) a short section on the magnetodynamics of fluids.

The general style of the book is quite unlike that of any book written at a comparable level in English. The French language perhaps lends itself to a degree of formality of expression that is now quite unusual in English texts on fluid mechanics; this formality of expression is combined with quite a high level of abstraction in the mathematical approach adopted, which presumably reflects the high level of preliminary mathematical training expected of students of mechanics at the École Polytechnique. On the other hand, there are frequent descriptions of practical devices (e.g. Pitot tube, anemometer, strain gauge, etc.) used in experiments, although the actual results of experiments are rarely recorded. The treatment is careful and scholarly and I could find very few statements that were positively wrong or misleading. One such is the statement on p. 72 that the effective point of application of the force between a jet and an inclined plane upon which it is incident is the point of intersection of the jet axis and the plane. Again, in the free-streamline problem of the jet emerging from a slit in a plane wall, figure 10 on p. 387 suggests that the free streamline suffers a discontinuity of slope at the salient edge of the slit; the slope is in fact continuous at this point (although the curvature of the streamline has an integrable singularity). But these are small points to quibble at in a text which contains a wealth of interesting and carefully considered material.

It would perhaps be inappropriate to attempt to review here Volume II of Mandel's work which is concerned wholly with solid mechanics. Suffice it to say that, while the main body of the text is devoted to linear elasticity, more than half the volume (!) is devoted to substantial appendices on more advanced topics, including general second-order theory (stress tensor a quadratic function of strain tensor), plasticity, and visco-elasticity. The impression of thorough scholarship is maintained throughout, and the scope and achievement of the author is nothing short of astonishing.

Gontier's work is similarly designed to cover a specific course on the Mechanics of Deformable Media given at the Faculté des Sciences in Lille. The level of the work is similar to that of Mandel but the scope is more limited. The book opens with a systematic development of Cartesian tensor algebra and analysis; this is admirably clear, though perhaps a little concise for the average student. This is followed by two chapters on the kinematics of deformation, including such esoteric topics as the material derivative of the Eulerian deformation tensor. Then there are two extremely thorough chapters on stress and the Cauchy equation and on the implications of isotropy for the constitutive relations. It is interesting to see that the author has thought it worth while to include a detailed discussion of the general Stokesian fluid in which the stress tensor $\boldsymbol{\sigma}$

is functionally related to the rate of strain tensor $\mathbf{e}$; he gives the proof of the remarkable result that this relation must be reducible to the form

$$\boldsymbol{\sigma} = (-p+a)\mathbf{I} + b\mathbf{e} + c\mathbf{e}^2$$

where a, b and c are functions of the invariants of $\mathbf{e}$ and of density and temperature, but he fails to note that the result is of merely academic interest (except in the linear (Newtonian) case) since, when non-linear terms are included in the relationship, the dependence of $\boldsymbol{\sigma}$ on the vorticity tensor must also for consistency be allowed for.

There is a curious contrast between the last two chapters of Gontier's book which arises out of the attempt to maintain generality on the one hand and to include topics of traditional fluid mechanical interest on the other. Chapter 8 consists of a treatment of tensor manipulations in curvilinear co-ordinates and the heavy apparatus of covariant and contravariant tensors, Christoffel symbols, etc., is set up. This formalism finds little application as far as Newtonian fluids are concerned, and none at all in the context of ideal fluids to which the final chapter is devoted. In this, we return to severely traditional topics governed by Laplace's equation, and the student who has worked conscientiously through chapter 8 may well be excused wondering what is the point of it all. General tensor analysis does of course find application in finite deformation theory in solid mechanics, but Gontier does not pursue the subject far enough to make the need at all convincing.

As far as fluids are concerned, Gontier devotes himself at one extreme to the general theory of constitutive relations, and at the other to the classical theory of ideal fluids (for the most part with vorticity). The treatment of these topics has a refreshing originality, and gives a great deal of food for thought (for the lecturer if not for the student). But there is almost complete omission of the vast range of topics, of central importance in modern fluid mechanics, in which ordinary Newtonian viscosity plays a crucial part.

A final comment on Gontier's book, which applies also to a lesser extent to that of Mandel, concerns the relative lack of what is normally described as the 'physical interpretation of mathematical results'. This is to some extent a matter of taste, and many would maintain that mathematics speaks for itself, and that physical discussion appended to a careful mathematical analysis is liable to obscure rather than to clarify the issues involved. There is a danger here that must be recognised—bad physical discussion is certainly worse than no discussion at all; but at the same time it is surely important to encourage the attempt (i) to draw certain simple conclusions from a mathematical analysis, no matter how complicated the latter may be, and (ii) to identify those physical processes which are revealed by the analysis to be of controlling importance. Only in this way can the student develop the physical insight that is so essential in finding an approach to more difficult problems that may not be immediately amenable to mathematical analysis, but that may succumb when the appropriate physical simplifying assumptions are made.

H. K. Moffatt